

- (1) H_0 : Subject choice is independent of Gender
 H_1 : Subject choice is associated with gender

(2) Expected

	B	C	F	P
M	5.45	33.32	28.48	35.75
F	3.55	21.68	18.52	23.25

(1) Observed

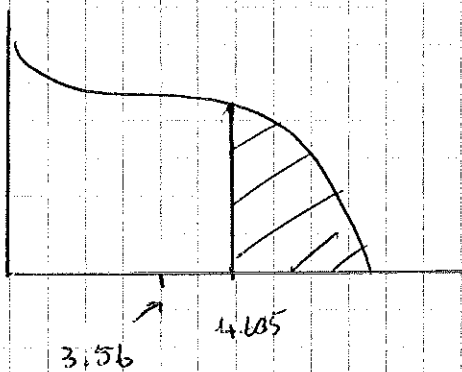
	B	C	F	P
M	7	31	25	40
F	2	24	22	19

(4) (55) (47) (59) (170)

Total $\chi^2 = 3.56 = \text{TEST STATISTIC}$

CRITICAL VALUE: $\chi^2_{(2)}(10\%) = 4.605$

$v = (3-1) \times (2-1) = 2$



(4) χ^2 $\frac{(O-E)^2}{E}$

	B	C	F	P+B
M	/	0.1615	0.4252	0.8165
F	/	0.2483	0.6539	1.2552

Must combine cells as expected < 5

Makes sense to combine Polish with Bulgarian

(3) New Combined Expected

	B	F	P+B
M	/	33.32	28.48
F	/	21.68	18.52

$4.605 > 3.56$

\therefore Accept H_0

Insufficient evidence at 10% level to suggest subject choice is associated with gender.

(2) a) $H_0: N = 8$

$H_1: N \neq 8$ (2 tailed test)

Data (from calc):

$n = 9$

$\sum x = 84$

$\sum x^2 = 840$

$\bar{x} = 9/3$

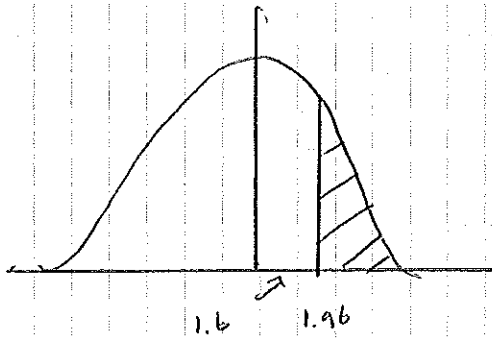
$\sigma = 2.5$

$\sigma^2 = 2.5^2$

As we know σ , we use z-distribution

TEST STATISTIC: $Z = \frac{9\frac{1}{3} - 8}{\frac{2.5}{\sqrt{9}}} = 1.6$

CRITICAL VALUE: 5% sig, 2 tailed = ± 1.96



$1.6 < 1.96$
 \therefore Accept H_0

Insufficient evidence at 5% level to suggest mean completion time has changed.

b) Neither error has occurred as we accepted $H_0: \mu = 8$ and $N = 8$!

③ a) i) $X \sim Po(3.6)$

$P(X \leq 3) = 0.5152$ (from tables)

ii) $Y \sim Po(4.4)$

$P(Y = 5) = \frac{e^{-4.4} \times (4.4)^5}{5!} = 0.1687...$

b) i) $T \sim Po(8)$

Assumption: X & Y are INDEPENDENT

ii) $P(6 < T < 12) = P(T \leq 11) - P(T \leq 6)$
 $= 0.8881 - 0.3134 = 0.575$

iii) $P(T > 14) = 1 - P(T \leq 14)$
 $= 1 - 0.9827 = 0.0173$

For 2 consecutive days = $0.0173^2 = 0.00029...$
 $= 0.0003$ (1sf)

iv) Need number so $P(T \leq n) \geq 0.99$

$P(T \leq 14) = 0.9827$ (from tables)

$P(T \leq 15) = 0.9918$

\therefore minimum number = 15.

$$\begin{aligned}
 (4) \quad a) \quad P(-3c/4 < X < 3c/4) &= F(3c/4) - F(-3c/4) \\
 &= \frac{3c/4 + c}{4c} - \frac{-3c/4 + c}{4c} \\
 &= \frac{3c/2}{4c} = \frac{3/2}{4} = 3/8
 \end{aligned}$$

b) for $f(x)$ need to differentiate $F(x)$

$$\rightarrow \text{if } y = \frac{xc + c}{4c} \rightarrow y = \frac{xc}{4c} + \frac{c}{4c}$$

$$\frac{dy}{dx} = \frac{1}{4c} + 0 \approx f(x)$$

For $xc > 3c & xc < -c$
we $F'(x) = 0$

$$\rightarrow f(x) = \begin{cases} 1/4c & -c \leq xc \leq 3c \\ 0 & \text{otherwise} \end{cases}$$

c) i) $E(X) = \frac{-c + 3c}{2} = c$

ii) $\text{Var}(X) = 1/2 (3c - (-c))^2 = 1/2 (4c)^2 = 4c^2/3$

(5) a) i) $\bar{x} = 1/2 (70.65 + 80.35) = 75.5$

ii) $\text{width} = \text{MAX} - \text{MIN} = 80.35 - 70.65 = 9.7$

iii) t distribution, $v = 16 - 1 = 15$, CRITICAL VALUE (98%) = 2.602

Standard error of mean = $\frac{s}{\sqrt{n}}$

98% CI $75.5 \pm 2.602 \frac{s}{\sqrt{n}}$

we know width = 9.7

$$\therefore 2.602 \frac{s}{\sqrt{n}} = \frac{9.7}{2}$$

$$\rightarrow \frac{s}{\sqrt{n}} = \frac{9.7}{2 \times 2.602} = 1.8639...$$

iv) $\frac{s}{\sqrt{n}} = 1.8639$ $n = 16$

$$\rightarrow s = 4 \times 1.8639$$

$$\rightarrow s^2 = (4 \times 1.8639)^2 = 55.5840... = 55.6 \quad (3sf)$$

= unbiased estimator of σ^2

b) 95% CI, t value = 2.131
 $\rightarrow 75.5 \pm 2.131 \times 1.8639 \leftarrow \frac{s}{\sqrt{n}}$
 $= 75.5 \pm 3.9719...$
 $= (71.5, 79.5)$

c) i) mean = 75.5, so must be 2.5g each side
 $\rightarrow (73, 78)$

ii) $75.5 \pm p \times 1.8639$

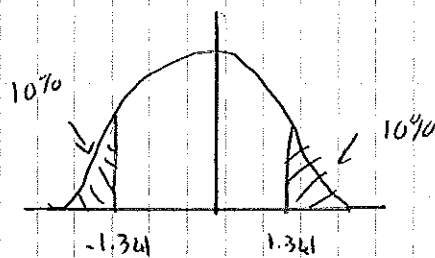
As width must be 5, $p \times 1.8639 = 2.5$

$\rightarrow p = \frac{2.5}{1.8639} = 1.341$

$n = 16, v = 16 - 1 = 15$

From tables, $P(X \leq 1.341) = 0.9$

As 2 tailed test (confidence interval), α must be 80%



6) a)

r	1	2	3	4
$P(R=r)$	$\frac{2}{3}$	$\frac{2}{9}$	$\frac{2}{27}$	K

As probs $\leq 1 \rightarrow \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + K = 1$

$\rightarrow K = \frac{1}{27}$

b) $P(R \geq 3) = P(R=3) + P(R=4)$
 $= \frac{2}{27} + \frac{1}{27} = \frac{1}{9}$

c) i) $E(L) = 27 \times E(R) + 5$

$E(R) = 1 \times \frac{2}{3} + 2 \times \frac{2}{9} + 3 \times \frac{2}{27} + 4 \times \frac{1}{27}$
 $= \frac{40}{27}$

$\rightarrow E(L) = 27 \times \frac{40}{27} + 5$
 $= 45$

$$i) E(R^2) = 1^2 \times \frac{2}{3} + 2^2 \times \frac{3}{4} + 3^2 \times \frac{2}{27} + 4^2 \times \frac{1}{27}$$

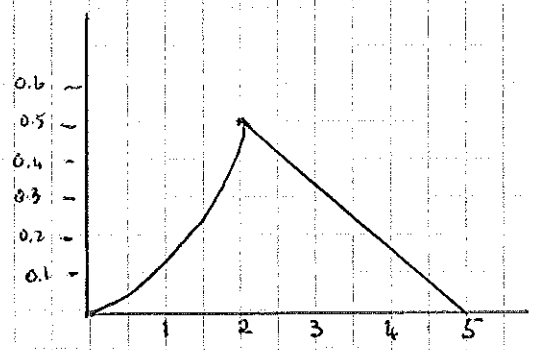
$$= \frac{76}{27}$$

$$\therefore \text{Var}(R) = \frac{76}{27} - \left(\frac{40}{27}\right)^2 = \frac{452}{729}$$

$$\text{SD}(R) = \sqrt{\frac{452}{729}}$$

$$\therefore \text{SD}(C) = 27 \times \sqrt{\frac{452}{729}} = 21.2602\dots$$

① d)



b) First need $F(2) = P(X \leq 2)$

$$= 1 - P(X \geq 2)$$

$$= 1 - \left(\frac{1}{2} \times 3 \times 0.5\right) = 0.25$$

↑ Triangle on diagram

For $2 < X < 5$

$$F(x) = F(2) + \int_2^x \frac{1}{6}(5-x) dx$$

$$= 0.25 + \frac{1}{6} \int_2^x 5-x dx$$

$$= 0.25 + \frac{1}{6} \left[5x - \frac{x^2}{2} \right]_2^x$$

$$= 0.25 + \frac{1}{6} \left[5x - \frac{x^2}{2} - 10 + 2 \right]$$

$$= 0.25 + \frac{5x}{6} - \frac{x^2}{12} - \frac{8}{6}$$

$$= \frac{x^2}{12} + \frac{5x}{6} - \frac{13}{12}$$

$$= -\frac{1}{12} (x^2 - 10x + 13)$$

$$= -\frac{1}{12} (x^2 - 10x + 25) + 1$$

$$= 1 - \frac{1}{12} (x-5)^2$$

$$= 1 - \frac{1}{12} (5-x)^2$$

$$c) P(X \geq 3 / X \leq 4) = \frac{P(A \cap B)}{P(B)}$$

From 511

$$P(B) = P(X \leq 4) = F(4) = 1 - \frac{1}{12}(5-4)^2 = \frac{11}{12}$$

$$P(A \cap B) = P(X \geq 3 \text{ AND } X \leq 4)$$

$$= P(3 \leq X \leq 4)$$

$$= F(4) - F(3)$$

$$= \frac{11}{12} - \left[1 - \frac{1}{12}(2)^2 \right] = \frac{11}{12} - \frac{2}{3}$$

$$= \frac{11}{12} - \frac{8}{12} = \frac{3}{12} = \frac{1}{4}$$

$$\therefore P(X \geq 3 / X \leq 4) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{11}{12}} = \frac{3}{11}$$